AP Calculus AB | Homework 4.4 | Name:
Dr. Paul L. Bailey | Tuesday, January 7, 2020

Write your homework *neatly*, *in pencil*, on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem*, or at least enough of it so that your work is readable. When appropriate, *write in sentences*.

The main theory of section 4.4 is summarized below. Note that our definition for "concave up" and "concave down" is more general than that in Thomas.

Definition 1. Let f be a function defined on an interval I. Let $x_1, x_2 \in I$ with $x_1 < x_2$. The *chord* from $(x_1, f(x_1))$ to $(x_2, f(x_2))$ is the function

$$k : [x_1, x_2] \to \mathbb{R} \quad \text{given by} \quad k(x) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) + f(x_1).$$

We say that f is *concave up on I* if $k(x) \geq f(x)$ for all $x_1, x_2 \in I$.
We say that f is *concave down on I* if $k(x) \leq f(x)$ for all $x_1, x_2 \in I$.

Thus, the graph of k is the line segment from $(x_1, f(x_1))$ to $(x_2, f(x_2))$; f is concave up if this line segment always lies above the graph of f, and f is concave down if this line segment always lies below the graph of f.

Definition 2. Let f be defined in an interval containing c.
We say that f has a *point of inflection* at c if the graph of f has a tangent line at c, and the concavity of f changes at c.

Theorem 1. (Second Derivative Test for Concavity)
Let f be twice differentiable at c and suppose $f'(c) = 0$.

- *If $f''(c) > 0$, then f has a local minimum at c.*
- *If $f''(c) < 0$, then f has a local maximum at c.*

Theorem 2. (Second Derivative Test for Local Extrema)
Let f be twice differentiable on an open interval I.

- *If $f'' > 0$ on I, then f is concave up on I.*
- *If $f'' < 0$ on I, then f is concave down on I.*

Problem 1. Let $f(x) = x^2 - 4x + 3$. Find intervals on which f is concave up and concave down.

Problem 2. Let $f(x) = x^3 - 3x + 3$. Find intervals on which f is concave up and concave down.

Problem 3. Let $f(x) = x^4 - 2x^2$. Find intervals on which f is concave up and concave down.

Problem 4. Let
$$f(x) = x^4 - 10x^2 + 9.$$

(a) Find intervals on which f is increasing and decreasing.

(b) Find intervals on which f is concave up and concave down.

Problem 5. Let
$$f(x) = \begin{cases} x - n & \text{if } x \in [n, n+1) \text{ for an even integer } n \\ n - x + 1 & \text{if } x \in [n, n+1) \text{ for an odd integer } n \end{cases}$$
Find intervals on which f is concave up and concave down.

Problem 6. Let

$$f : [0, 4\pi] \to \mathbb{R} \quad \text{be given by} \quad f(x) = \sin x.$$

(a) Find all critical points of f.

(b) Find all points of inflection of f.

(c) Find intervals on which f is increasing and decreasing.

(d) Find intervals on which f is concave up and concave down.

Problem 7. The *change of base* formula for logarithms is $\log_b x = \dfrac{\log_a x}{\log_a b}$. Use this to compute $\dfrac{d}{dx} \log_{10} x$.

Problem 8. Let

$$f(x) = \frac{e^x + e^{-x}}{2} \quad \text{and} \quad g(x) = \frac{e^x - e^{-x}}{2}.$$

(a) Show that $f'(x) = g(x)$ and $g'(x) = f(x)$, for all $x \in \mathbb{R}$.

(b) Show that $f(2x) = f(x)^2 - g(x)^2$.

Problem 9. Consider the cubic polynomial

$$f(x) = x^3 + ax^2 + bx.$$

(a) Find the values of a and b for which f has two zeros.

(b) Find the values of a and b for which f has two local extrema.

(c) Find the values of a and b for which f has exactly one horizontal tangent.

(d) Find the values of a and b for which f has no horizontal asymptotes.

Problem 10 (Thomas §3.5 # 60)**.** Suppose that the functions f and g and their derivatives with respect to x have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	1/3
1	3	-4	-1/3	-8/3

Find the derivatives with respect to x of the following combinations at the given value of x.

(a) $5f(x) - g(x)$, $x = 1$

(b) $f(x)g^3(x)$, $x = 0$

(c) $\dfrac{f(x)}{g(x) + 1}$, $x = 1$

(d) $f(g(x))$, $x = 0$

(e) $g(f(x))$, $x = 0$

(f) $(x^{11} + f(x))^{-2}$, $x = 1$

(g) $f(x + g(x))$, $x = 0$